Mesh Generation for Modeling and Simulation of Carbon Sequestration Processes

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Overview

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5. A linear CDT algorithm
6. Constraint Voronoi Meshing
Motivation

• Generating tessellations for simulation of propagating cracks in disordered media (geo materials, concrete, polycrystalline materials ... etc.)

• Probability of seeing a straight crack propagate through a random field is zero.
Eliminating mesh induced crack bias

- If cracks can grow only at element edges, then need to eliminate any directional bias in crack growth.
- Structured grids can result in strong mesh induced bias (nonobjective).
- need to use ‘random’ discretizations using Voronoi mesh.
- statistically isotropic tessellation.
Objectives

- A bias-free point cloud to minimize the effect of the final mesh on a crack propagation.
- A maximal distribution so the domain is saturated with the generated points.
- A minimum distance between the generated points.
- The ability to handle non-convex domains with multiple fractures, and/or holes.
- Running time should be linear in the number of generated points.
- Utilized memory should be as small as possible.
- Can be easily implemented in parallel.
- The output mesh should be conforming.
Computational Approach

- Solve the maximal Poisson-disk sampling problem.
- Extend that solution to non-convex domains.
- Generate the edge connectivity based on a new Constrained Delaunay triangulation method.
- Construct the corresponding Constrained Voronoi Diagram by retrieving the dual grid of a Delaunay mesh.
- Adjust the tessellation along internal boundaries to generate a conforming mesh.
- The proposed algorithm should be extended to 3D easily.
- The proposed algorithm should be extended to the non-uniform case easily.
Three conditions to be satisfied:

- Each point is a center of a disk, with radius $r$, that contains no other points:

  $$\forall x_i \in x_j \in X, x_i \neq x_j : ||x_i - x_j|| \geq r$$

- The point distribution should be bias-free:

  $$\forall x_i \in X, \forall \Omega \subset D : P(x_i \in \Omega) = \int_\Omega d\omega$$

- Termination is achieved when the domain is completely saturated:

  $$\forall x \in D, \exists x_i \in X : ||x - x_i|| < r$$
Challenges

- An efficient method to retrieve conflicts.
- Filling the small gaps between the disks.
- Detection of the termination condition.
The maximal Poisson-disk sampling

Our solution

- Utilizing a cartesian background grid
- Dynamic linear representation of the voids in the domain.
Voronoi Meshing for Simulation of Propagating Cracks

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CDT
CVM

The maximal Poisson-disk sampling: Results
The maximal Poisson-disk sampling: Results

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The maximal Poisson-disk sampling: Results
Our solution

- A maximal Poisson-disk sampling results in an upper bound for the edge length in the associated Delaunay tessellation.
- The background grid is utilized to retrieve possible neighbors which are filtered locally based on edge constraints and the Delaunay principal.
- This results in a linear algorithm capable of processing 250,000 points/second.
- The speed of the implementation is not sensitive to the shape of the domain or the number of the edge constraints.
- Angle bounds are $30^\circ - 120^\circ$ while edge length bounds are $r - 2r$. 
A linear CDT algorithm
Our linear CDT algorithm: Results
Our linear CDT algorithm: Results
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Our linear CDT algorithm: Results
Our solution

- After retrieving the constraint edge connectivity, the Voronoi mesh is constructed by retrieving the dual grid.
- A circum-center that results in a non-conforming Voronoi mesh adjacent to internal crack is repositioned to the mid-point of its nearest edge.
- Short edges are collapsed on the fly to ensure a minimum bound for the edges in the Final Voronoi mesh.
Constraint Voronoi Meshing: Results
Constraint Voronoi Meshing: Results

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[Images of Voronoi mesh patterns]
Constraint Voronoi Meshing: Results

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Thank you!